WHY BENFORD'S LAW WORKS FOR FUNCTION POINT ANALYSIS

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Introduction

Benford's Law is named for the late Dr. Frank Benford, a physicist formerly at the General Electric Company. In 1938, Dr. Benford noticed that pages of logarithms corresponding to numbers starting with the numeral 1 were much dirtier and more worn than other pages. (Browne 1998) In fact, in numerous lists of numbers he then studied from many real-life sources of data, the leading digit 1 occurred more often than the others, namely about 30% of the time.

A lively account of Benford's Law is from a blog entitled "Fabulous Adventures in Coding."

> While I was poking through my old numeric analysis textbooks to refresh my memory for this series on floating point arithmetic, I came across one of my favorite weird facts about math. A nonzero base-ten integer starts with some digit other than zero. You might naively expect that given a bunch of "random" numbers, you'd see every digit from 1 to 9 about equally often. You'd see as many 2's as 9's. You'd see each digit as the leading digit about 11% of the time. For example, consider a random integer between 100000 and 999999. One ninth begin with 1, one ninth begin with 2, etc. But in real-life datasets, that's not the case at all. If you just start grabbing thousands or millions of "random" numbers from newspapers and magazines and books, you soon see that about 30% of the numbers begin with 1, and it falls off rapidly from there. About 18% begin with 2, all the way down to less than 5% for 9. This oddity was discovered by Newcomb in 1881, and then rediscovered by Frank Benford, a physicist, in 1937. As often is the case, the fact became associated with the second discoverer and is now known as Benford's Law. Benford's Law has lots of practical applications. For instance, people who just make up numbers wholesale on their tax returns tend to pick "average seeming" numbers, and to humans, "average seeming" means "starts with a five." People think, I want something between $1000 and $10000, let's say, $5624. The IRS routinely scans tax returns to find unusually high percentages of leading 5's and examines those more carefully. Benford's result was carefully studied by many statisticians and other mathematicians, and we now have a multi-base form of the law.

> Given a bunch of numbers in base B, we'd expect to see leading digit n approximately in (1 + 1/n) / in B of the time. But what could possibly explain Benford's Law? (Lippert 2005)

This paper explains why Benford's Law works (at least in many situations), and shows that it also applies to function points. This paper is based on the previous research of the authors (Tichenor, Davis 2008).

Logarithms

The first stop on the way to understanding why Benford's Law works is to refresh our minds about exponents and logarithms (usually abbreviated as "log").

We are all familiar with how to express numbers using exponents. For example,

$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$

and so on, where the 1, 2, and 3 are exponents. However, we can then reverse-engineer these equations and say that

log 10 = 1
log 100 = 2
log 1000 = 3

and so on.

Any number can be expressed as 10 to some power. For example,

$10^{.3010} = 1$
$10^{.4771} = 2$
$10^{.6021} = 3$

and so on. You can check these with a scientific calculator. We can also reverse-engineer these equations and say that

log 1 = .3010
log 2 = .4771
log 3 = .6021

and so on.

Weber-Fechner Law

This study of the underlying causes of Benford's Law includes the research of Ernst Heinrich Weber. Weber (Wikipedia 2005) found a form of the law of diminishing returns relationship in humans between stimulus and response: as stimulus increased, response also increased but at a decreasing rate that is logarithmic. For example, if stimulus increased by a factor of 2 (i.e., 100%), then response increased by log 2, or .3010. If stimulus increased by a factor of 3, then response increased by a factor of log 3, or .4771. If stimulus increased from a factor of 2 to a factor of 3, then response increased by log 3 – log 2, or .4771 - .3010, or .1761. (Using this line of reasoning, if a stimulus level "increases" by a factor of 1, then there actually is no change in stimulus level and therefore no response.) This important finding is summarized in the below table, and was verified by Sinn (Sinn 2002).

Table 1. Weber-Fechner Law of Stimulus and Response

Stimulus Level	**Response Level Log**	**Incremental Response Increase from Previous Response Level**	**% Incremental Response Increase**
(1)	0	0	0
2	0.3010	0.3010	30.10%
3	0.4771	0.1761	17.61%
4	0.6021	0.1249	12.49%
5	0.6990	0.0969	9.69%
6	0.7782	0.0792	7.92%
7	0.8451	0.0669	6.69%
8	0.9031	0.0580	5.80%
9	0.9542	0.0512	5.12%
10	1.000	0.0458	4.58%

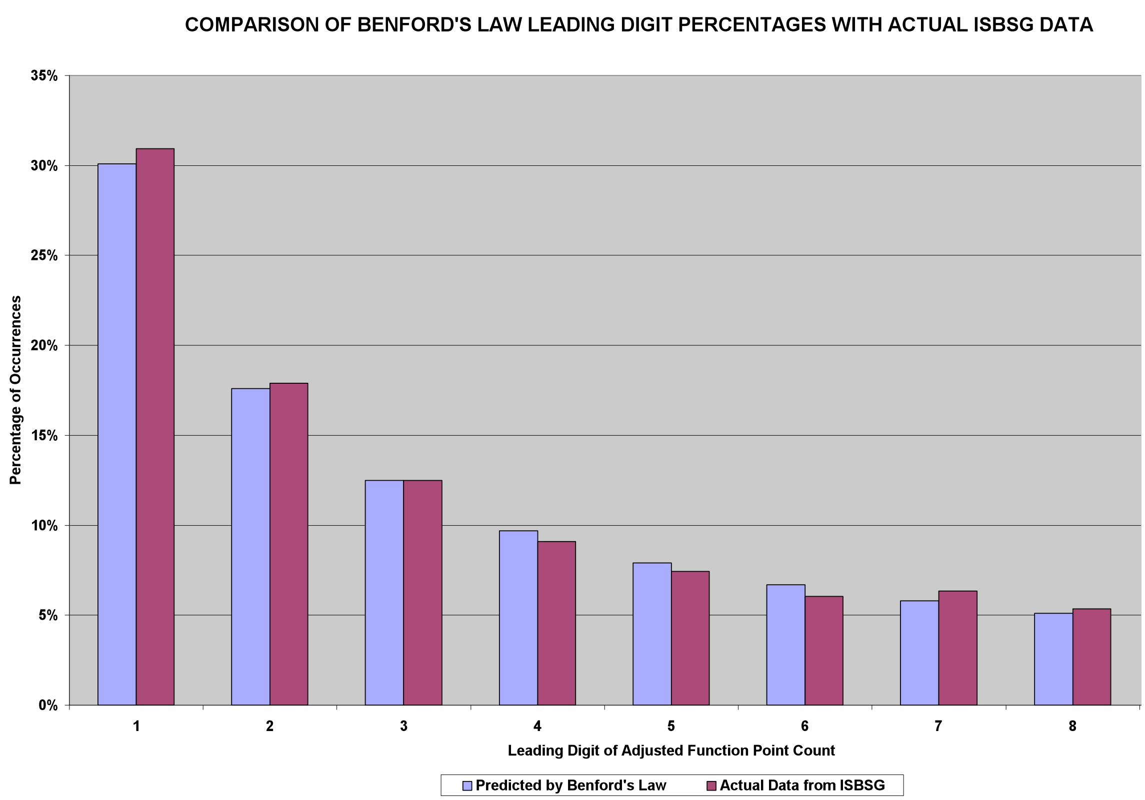
Now, it is unlikely that a stimulus level will increase from 1 to *exactly* 2. It could increase to any of numerous intermediate levels, say, 1.04, 1.3 or to 1.72. What is important here is that 30.10% (.3010) of the possible stimulus levels will be *from* 1 to 2. Put another way, those leading digits will be a 1 (such as the 1.04, 1.3 or 1.72). In the same way, stimulus levels could be 2.3, 2.47, or 2.989. The number of stimulus levels that will have a leading digit of 2 will be 17.61%, or .1761.

There is a sidebar that we also need to discuss. If the stimulus level increases from 1 to 2, then the response increases by 30.10%. Those familiar with logarithm math would conclude that it is also true that if the stimulus increases from 10 to 20, 100 to 200, 1000 to 2000, etc., then the corresponding responses would also have to increase by factors of 30.10%. What is important here is the following conclusion. According to the Weber-Fechner Law, if we randomly sample a statistically large number of responses, we will find that about 30.10% of them will have a stimulus level starting with a leading digit of 1. We will find that about 17.61% of the responses will have a corresponding stimulus level starting with a 2, and so on. We will find that only 4.58% will have a leading digit of 9.

How Benford's Law Applies to Function Points

Suppose that software development is a human stimulus and response activity. The response is customer satisfaction, and the stimulus is the amount of functionality recognized by the user. We can measure this amount of functionality by using the adjusted function point count methodology as defined by the International Function Point Users Group (IFPUG) Counting Practices Manual. If we sample a statistically large number of these function point counts, then we must find that about 30.10% of them have a leading digit of 1, about 17.71% must have a leading digit of 2, and so on through the leading digit of 9 – which should occur about 4.58% of the time. Benford's Law must apply.

To test this, we went into Release 10 of the International Software Benchmarking Standards Group (ISBSG) function point data and downloaded all 3,103 IFPUG adjusted function point counts that ISBSG believed rated in their count quality level of either A or B. (ISBSG 2008) Below is a graph of the results.



The percentages of responses having the leading digits distribution predicted by Benford's Law, and implied by the Weber-Fechner Law, is statistically the same as found in the ISBSG IFPUG adjusted function point data.

Conclusion

The agreement of the ISBSG data with Benford's law is statistically significant. Every deviation of the actual percentage in ISBSG data and that predicted by Benford's Law is less than 1%. We therefore conclude that software development is a stimulus and response activity, and that it is modeled almost perfectly using Benford's Law; if a customer desires 30.10% more customer satisfaction from software functionality, then the function point count must therefore be doubled. We also find very good reason to believe that IFPUG adjusted function point methodology is an outstanding measure of software functionality, and that the set of A and B IFPUG adjusted function point count data in ISBSG Release 10 is of outstanding quality.

REFERENCES

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BIOGRAPHY

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